Digital Image Processing

Lecture 8. Filtering in the Frequency Domain

Fall 2010

Outline

- Filtering in Fourier Transform Domain
- Homomorphic Filtering

Image Sharpening Using Frequency Domain Filters

A highpass filter is obtained from a given lowpass filter using

$$H_{HP}(u,v) = 1 - H_{LP}(u,v)$$

A 2-D ideal highpass filter (IHPL) is defined as

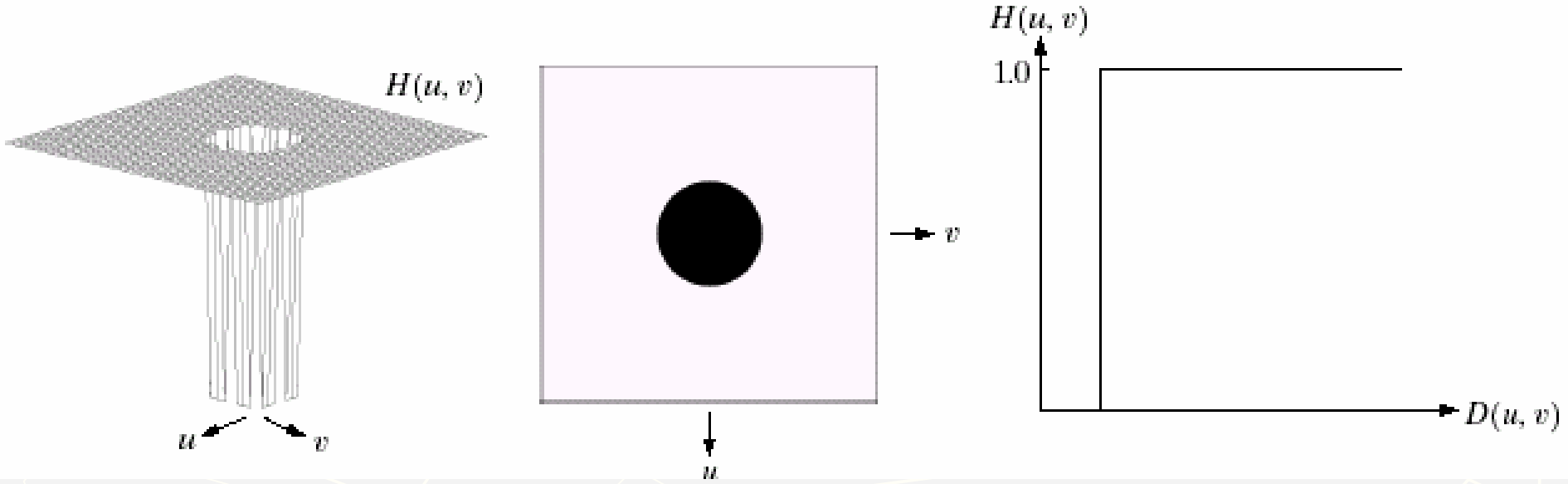
$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Ideal Highpass Filters

Ideal LPF Filter Transfer function

$$H(u,v) = \begin{cases} 0 & D(u,v) \le D_0 \\ 1 & D(u,v) > D_0 \end{cases}$$

where $D(u,v)$ = Distance from (u,v) to the center of the mask.

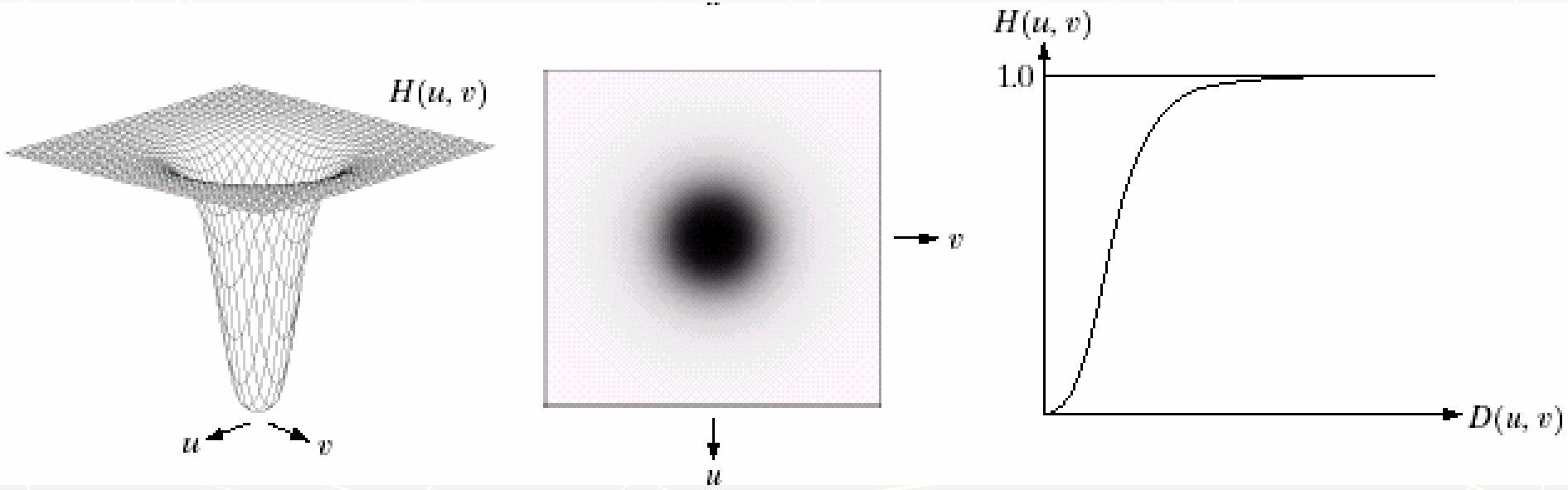


Butterworth Highpass Filters

Transfer function

$$H(u,v) = \frac{1}{1 + \left[D_0 / D(u,v)\right]^{2N}}$$

Where D_0 = Cut off frequency, N = filter order.

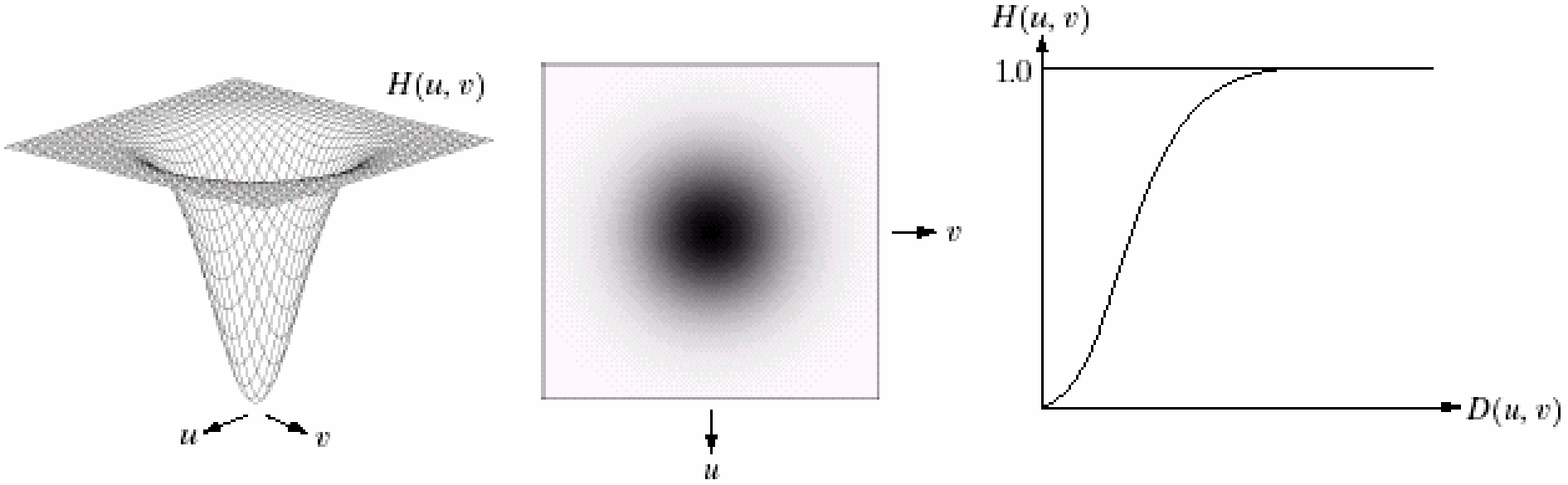


Gaussian Highpass Filters

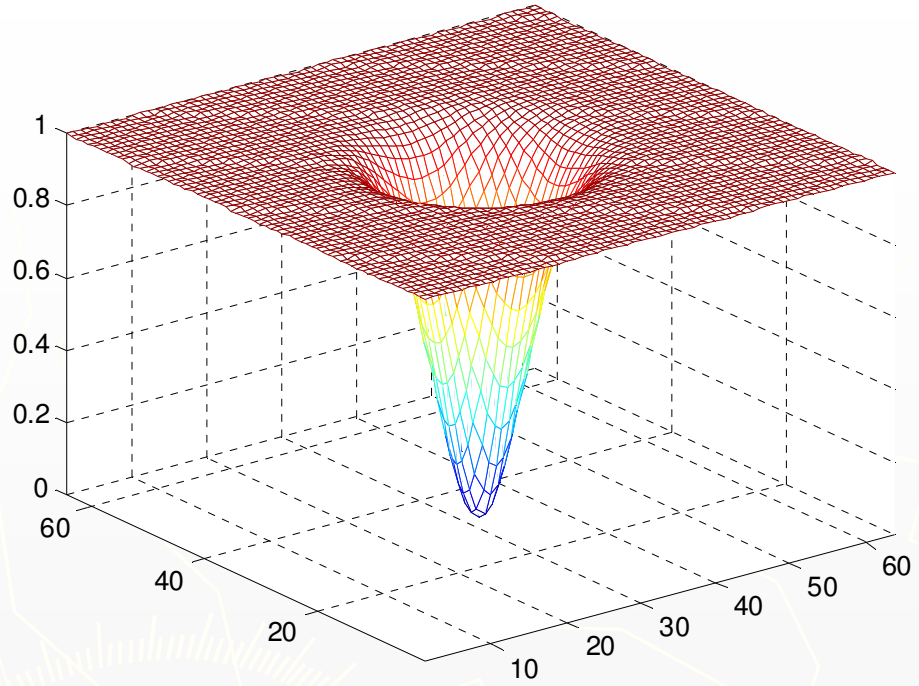
Transfer function

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Where D_0 = spread factor.



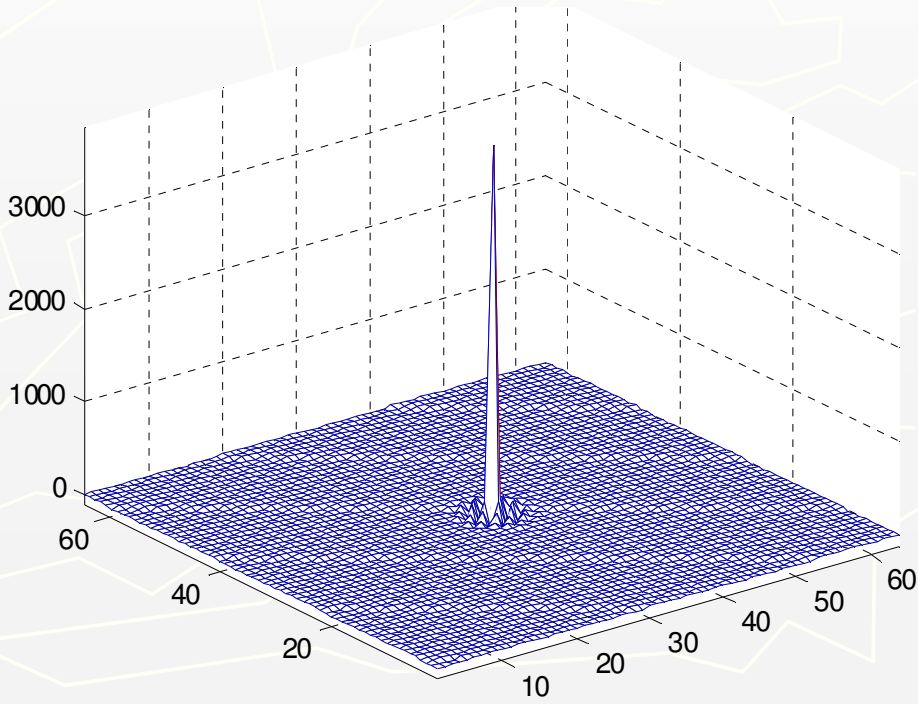
Gaussian Highpass Filters (cont.)



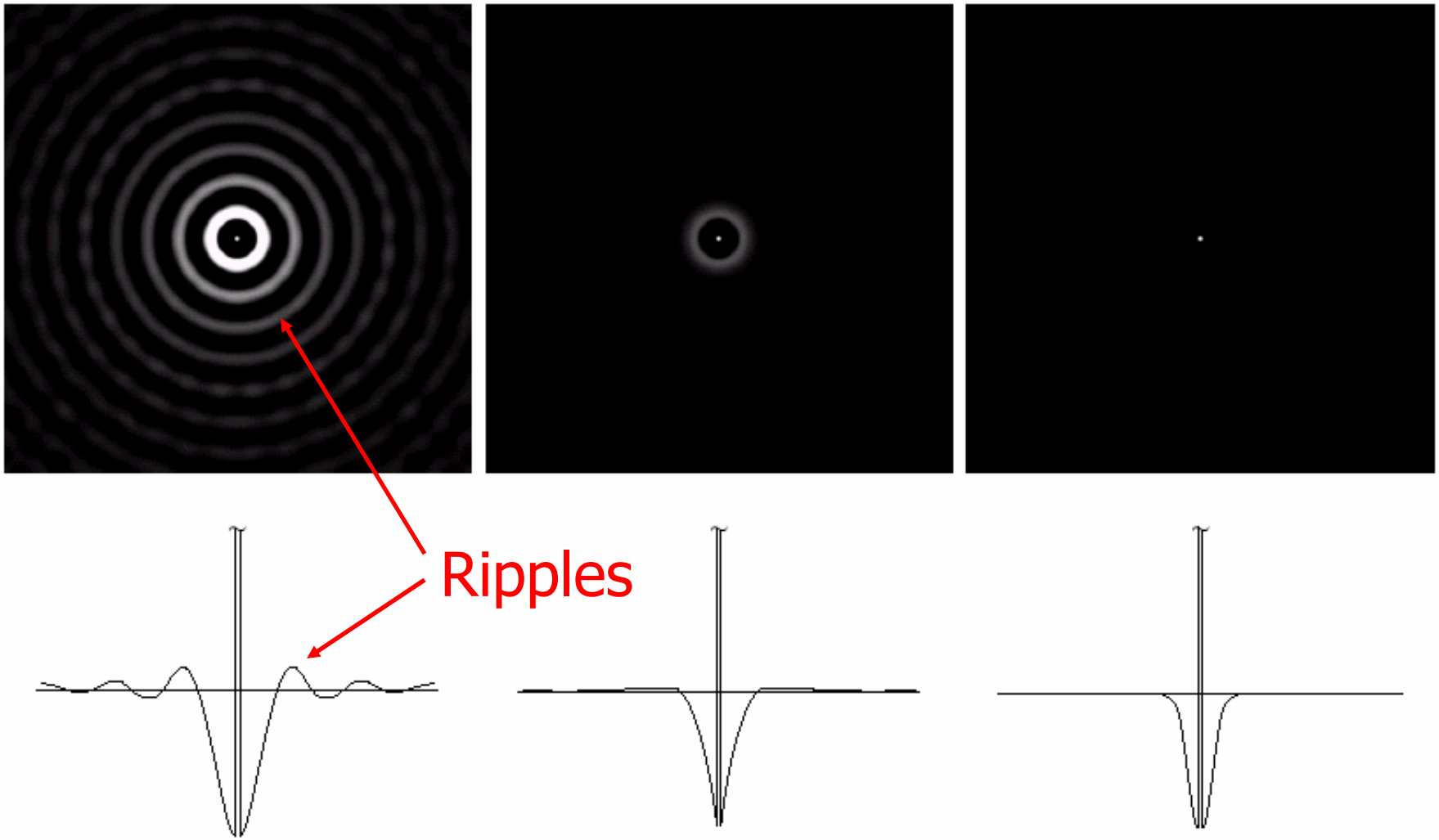
$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

Gaussian highpass filter with $D_0 = 5$

Spatial respones of the Gaussian highpass filter with $D_0 = 5$



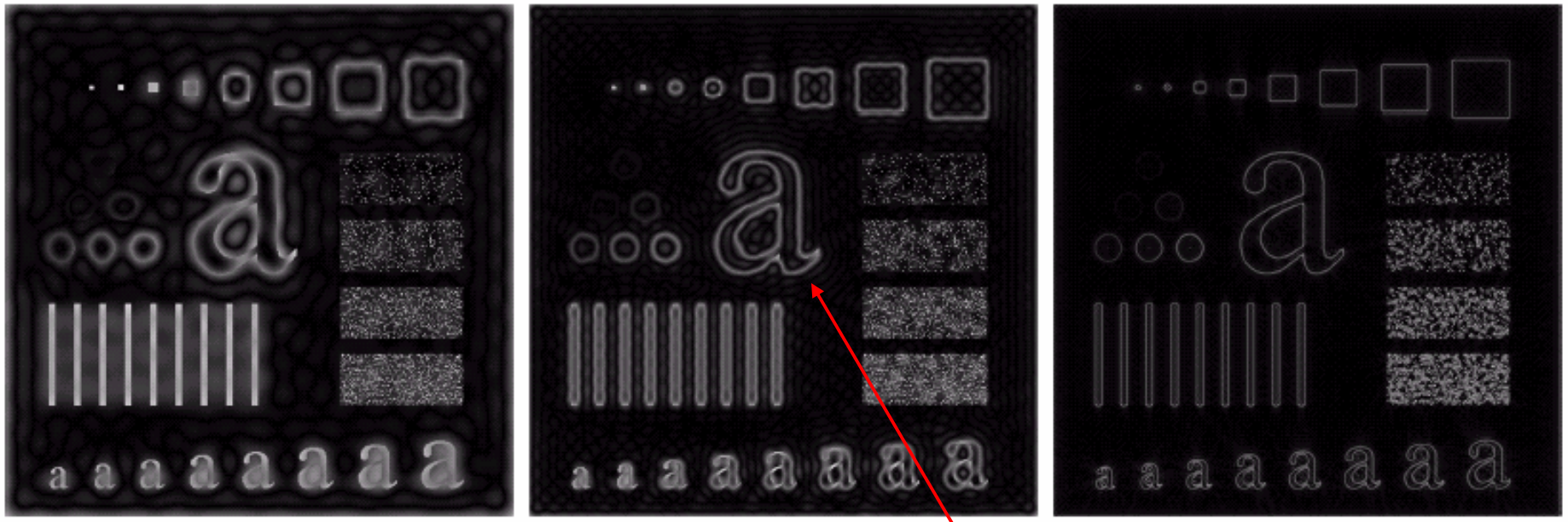
Spatial Responses of Highpass Filters



a b c

FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

Results of Ideal Highpass Filters

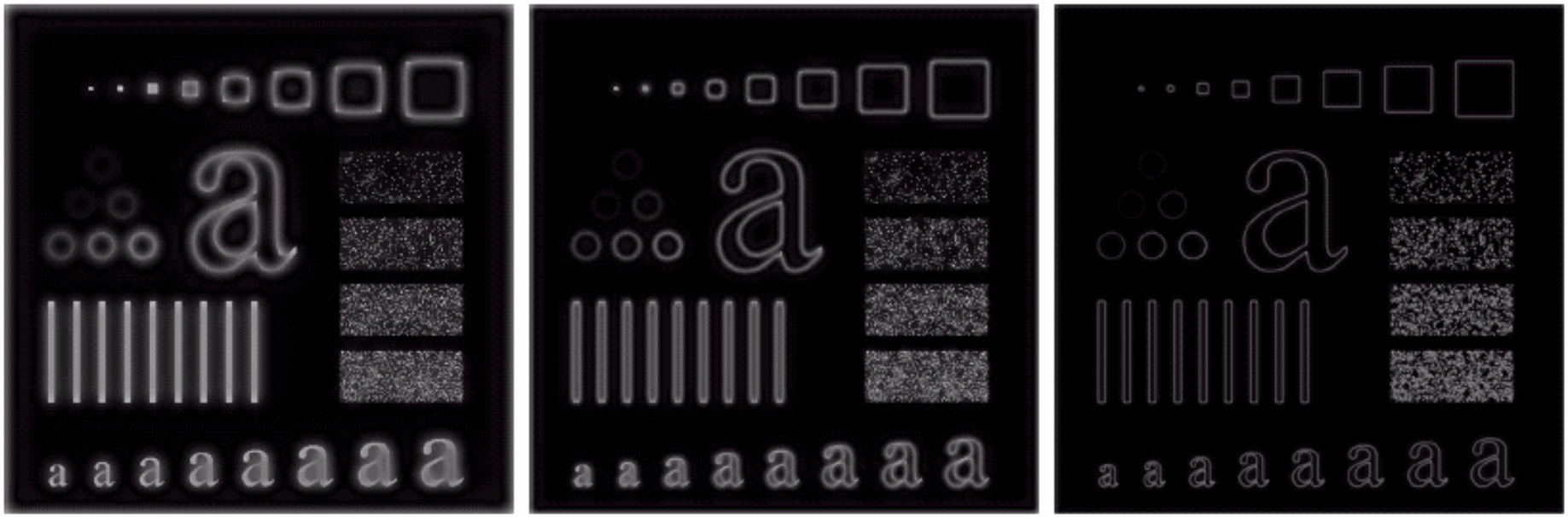


a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

Ringing effect can be obviously seen!

Results of Butterworth Highpass Filters



a b c

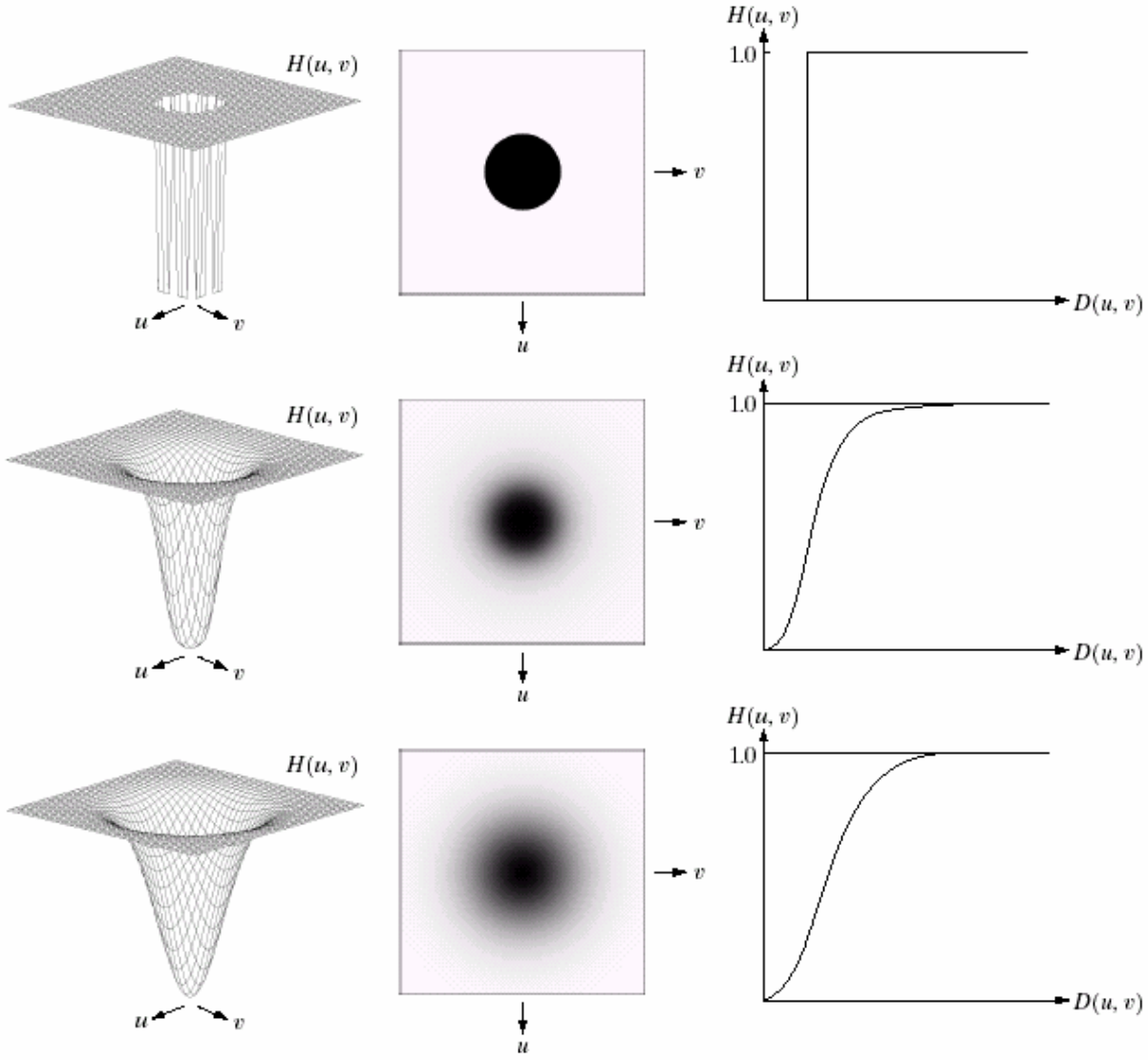
FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Results of Gaussian Highpass Filters

a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Highpass Filters

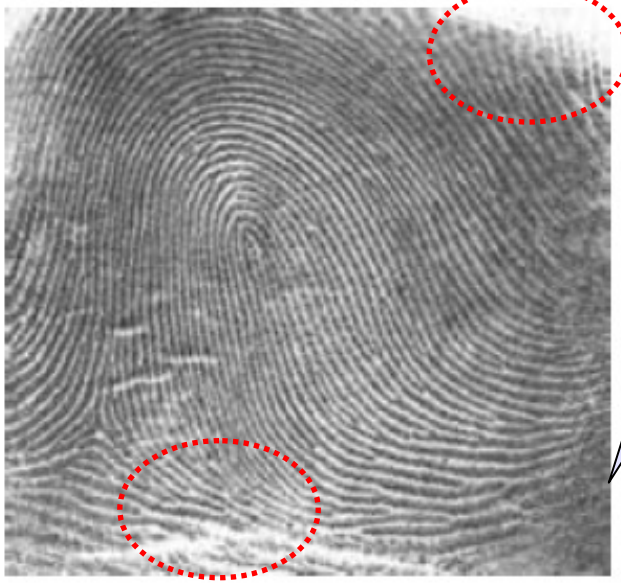


a b c
d e f
g h i

FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

$$H_{hp} = 1 - H_{lp}$$

Using Highpass Filtering and Threshold for Image Enhancement



BHPF
(order 4 with a cutoff frequency 50)

a b c

FIGURE 4.57 (a) Thumb print. (b) Result of highpass filtering (a). (c) Result of thresholding (b). (Original image courtesy of the U.S. National Institute of Standards and Technology.)

Laplacian Filter in the Frequency Domain

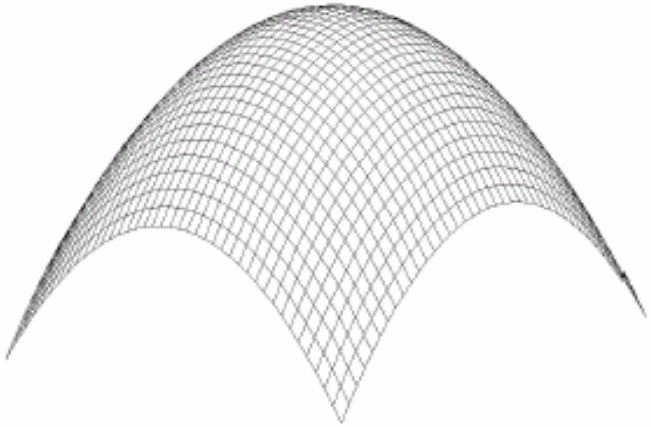
From Fourier Tr. Property: $\frac{d^n f(x)}{dx^n} \Leftrightarrow (ju)^n F(u)$

Then for Laplacian operator

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \Leftrightarrow -\left(u^2 + v^2\right) F(u,v)$$

We get

$$\nabla^2 \Leftrightarrow -\left(u^2 + v^2\right)$$



Surface plot

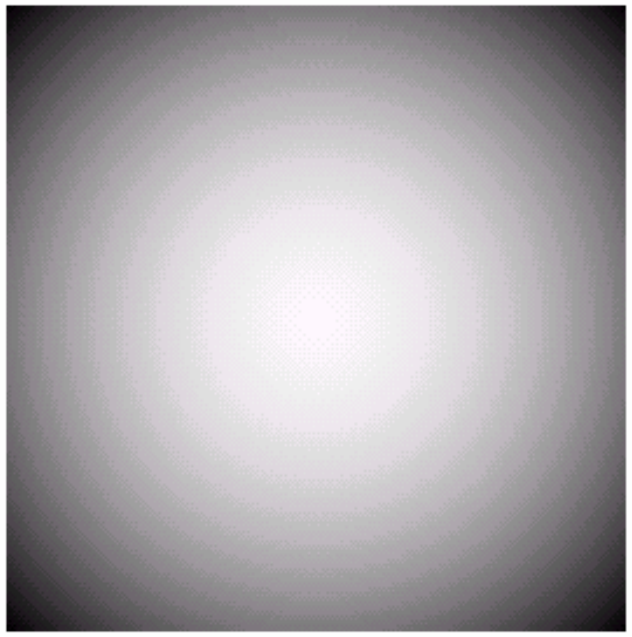
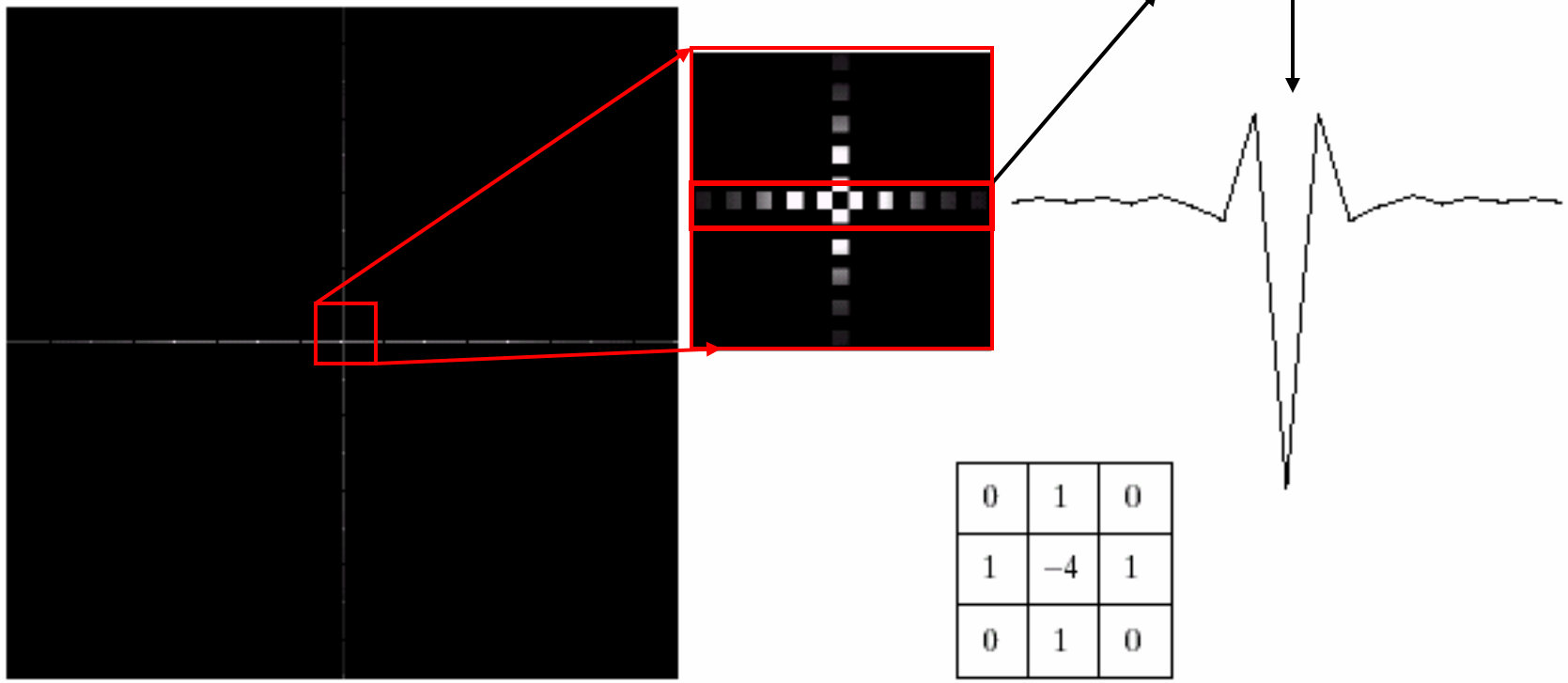


Image of $-(u^2+v^2)$

Laplacian Filter in the Frequency Domain (cont.)

Spatial response of $-(u^2+v^2)$



0	1	0
1	−4	1
0	1	0

Laplacian mask in Chapter 3

Sharpening Filtering in the Frequency Domain

Spatial Domain

$$f_{hp}(x,y) = f(x,y) - f_{lp}(x,y)$$

$$f_{hb}(x,y) = Af(x,y) - f_{lp}(x,y)$$

$$f_{hb}(x,y) = (A-1)f(x,y) + f(x,y) - f_{lp}(x,y)$$

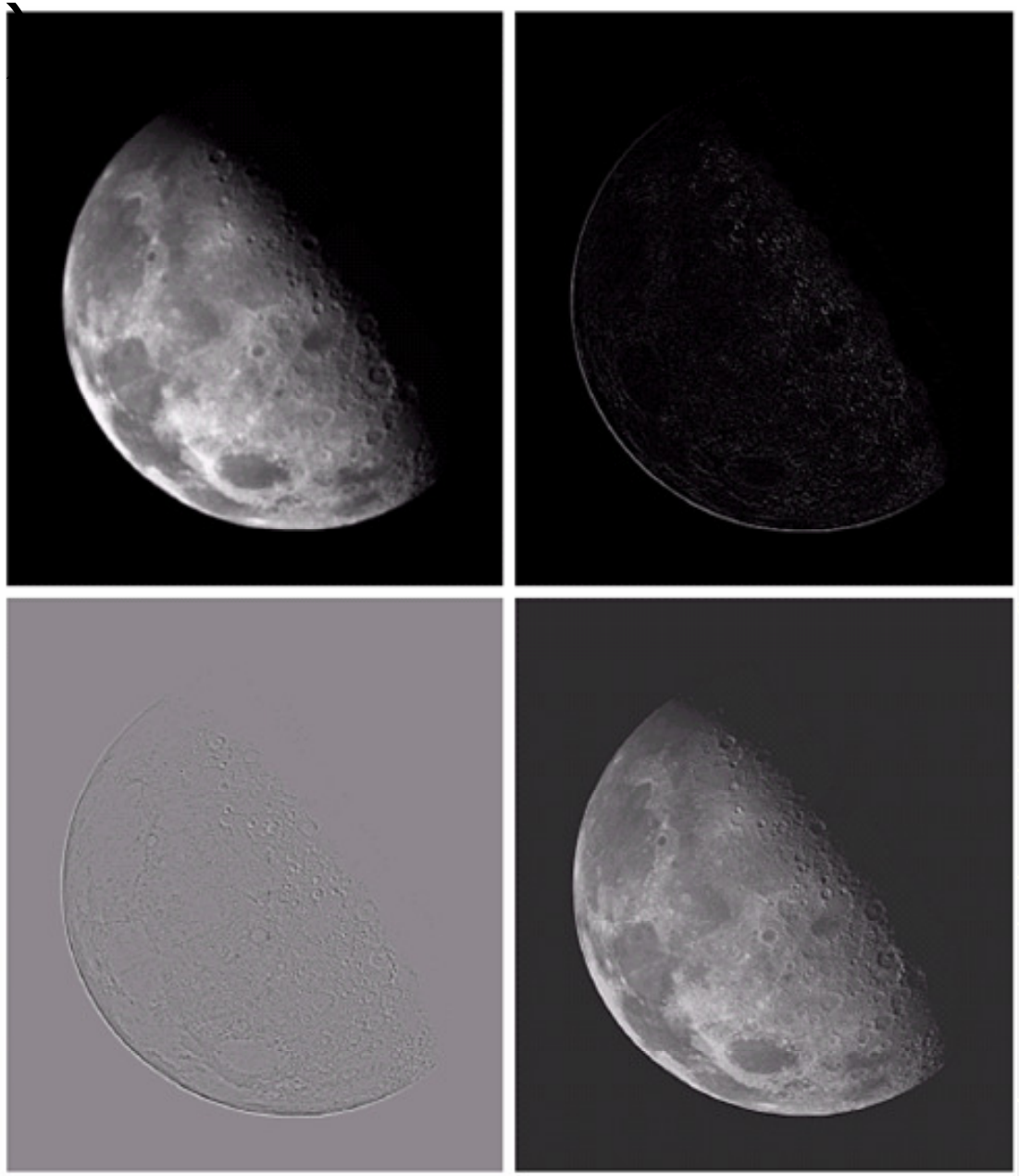
$$f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y)$$

Frequency Domain Filter

$$H_{hp}(u,v) = 1 - H_{lp}(u,v)$$

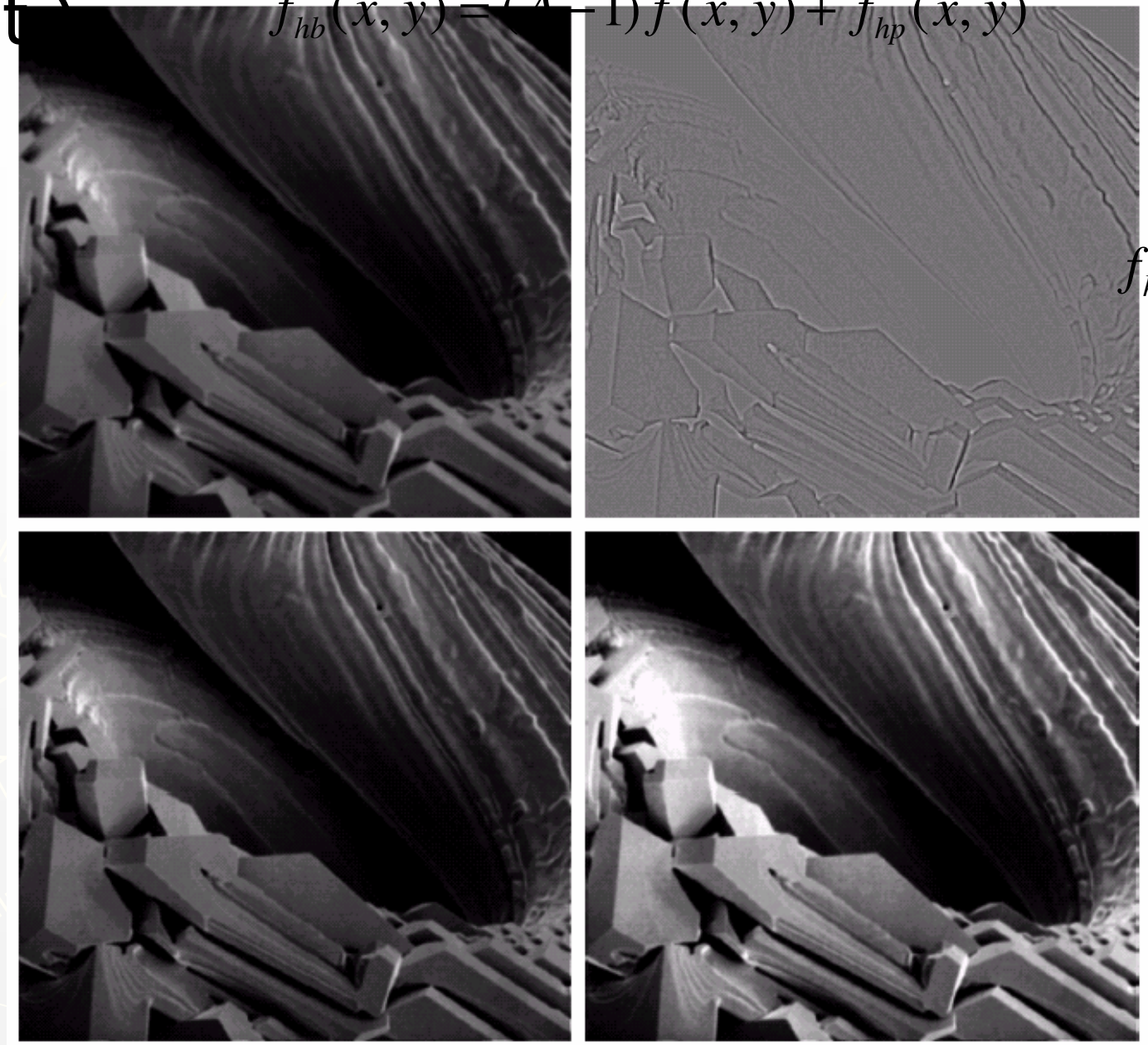
$$H_{hb}(u,v) = (A-1) + H_{hp}(u,v)$$

Sharpening Filtering in the Frequency Domain (cont.)



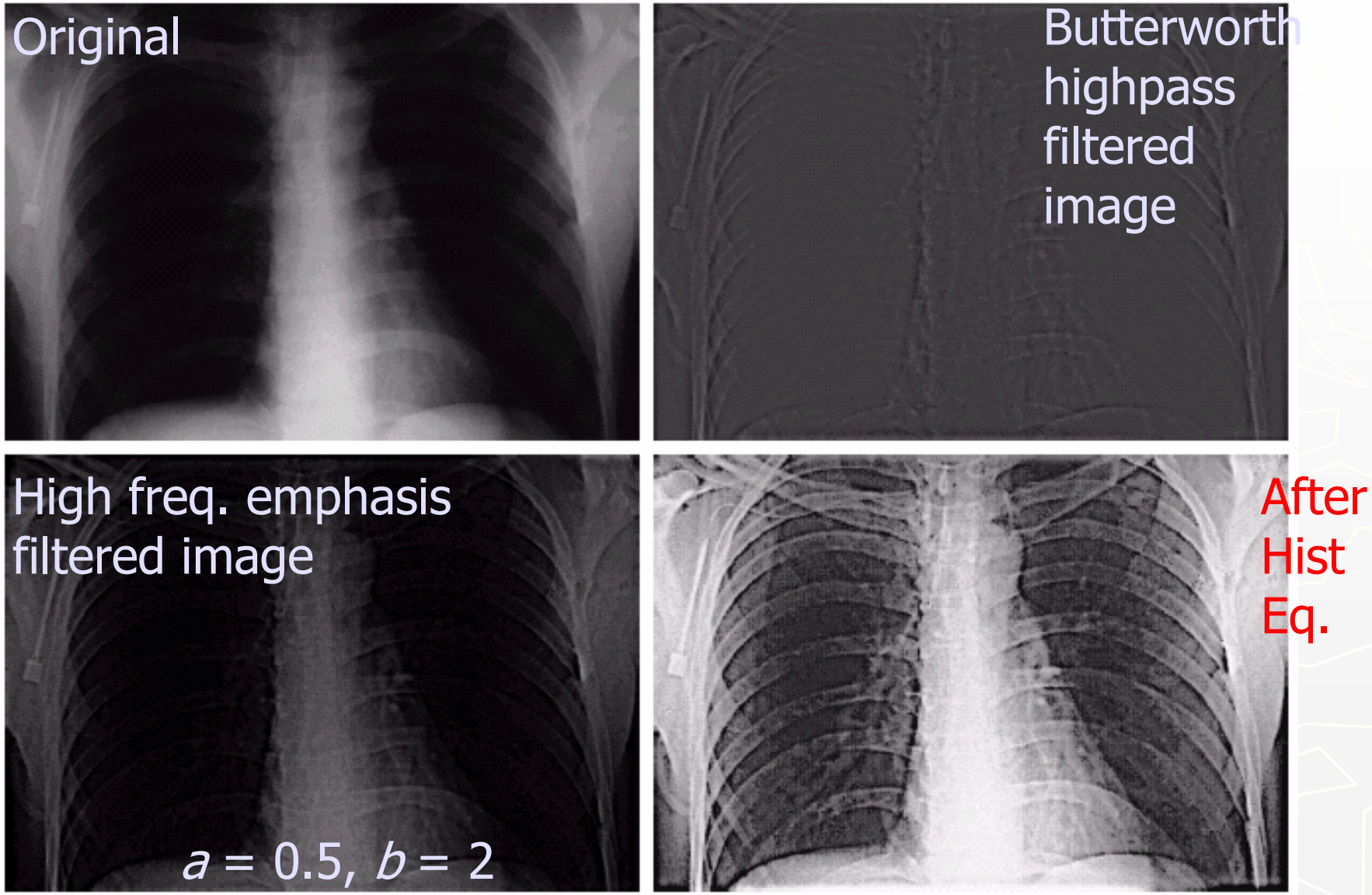
Sharpening Filtering in the Frequency Domain (cont.)

$$f_{hb}(x,y) = (A-1)f(x,y) + f_{hp}(x,y)$$



High Frequency Emphasis Filtering

$$H_{hfe}(u,v) = a + bH_{hp}(u,v)$$



Homomorphic Filtering

$$f(x,y) = i(x,y)r(x,y)$$

$$\Im\left[f(x,y)\right] = \Im\left[i(x,y)\right]\Im\left[r(x,y)\right] \; ?$$

$$z(x,y) = \ln f(x,y) = \ln i(x,y) + \ln r(x,y)$$

$$\Im\{z(x,y)\} = \Im\{\ln f(x,y)\} = \Im\{\ln i(x,y)\} + \Im\{\ln r(x,y)\}$$

$$Z(u,v) = F_i(u,v) + F_r(u,v)$$

Homomorphic Filtering

$$S(u,v) = H(u,v)Z(u,v)$$
$$= H(u,v)F_i(u,v) + H(u,v)F_r(u,v)$$
$$s(x,y) = \Im^{-1}\{S(u,v)\}$$
$$= \Im^{-1}\{H(u,v)F_i(u,v) + H(u,v)F_r(u,v)\}$$
$$= \Im^{-1}\{H(u,v)F_i(u,v)\} + \Im^{-1}\{H(u,v)F_r(u,v)\}$$
$$= i'(x,y) + r'(x,y)$$

$$g(x,y) = e^{s(x,y)} = e^{i'(x,y)}e^{r'(x,y)} = i_0(x,y)r_0(x,y)$$

Homomorphic Filtering

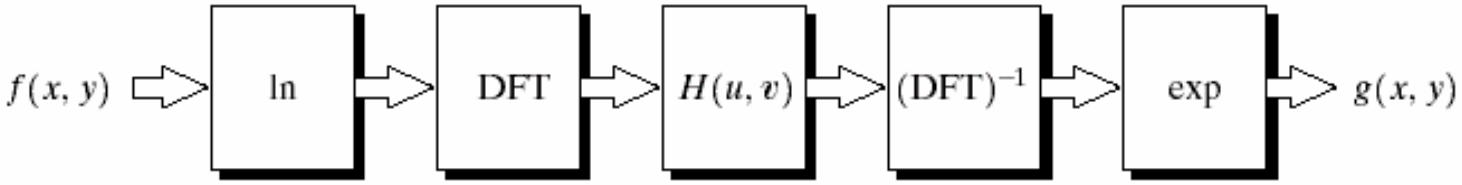


FIGURE 4.31 Homomorphic filtering approach for image enhancement.

The illumination component of an image generally is characterized by slow spatial variations, while the reflectance component tends to vary abruptly

These characteristics lead to associating the low frequencies of the Fourier transform of the logarithm of an image with illumination the high frequencies with reflectance.

Homomorphic Filtering

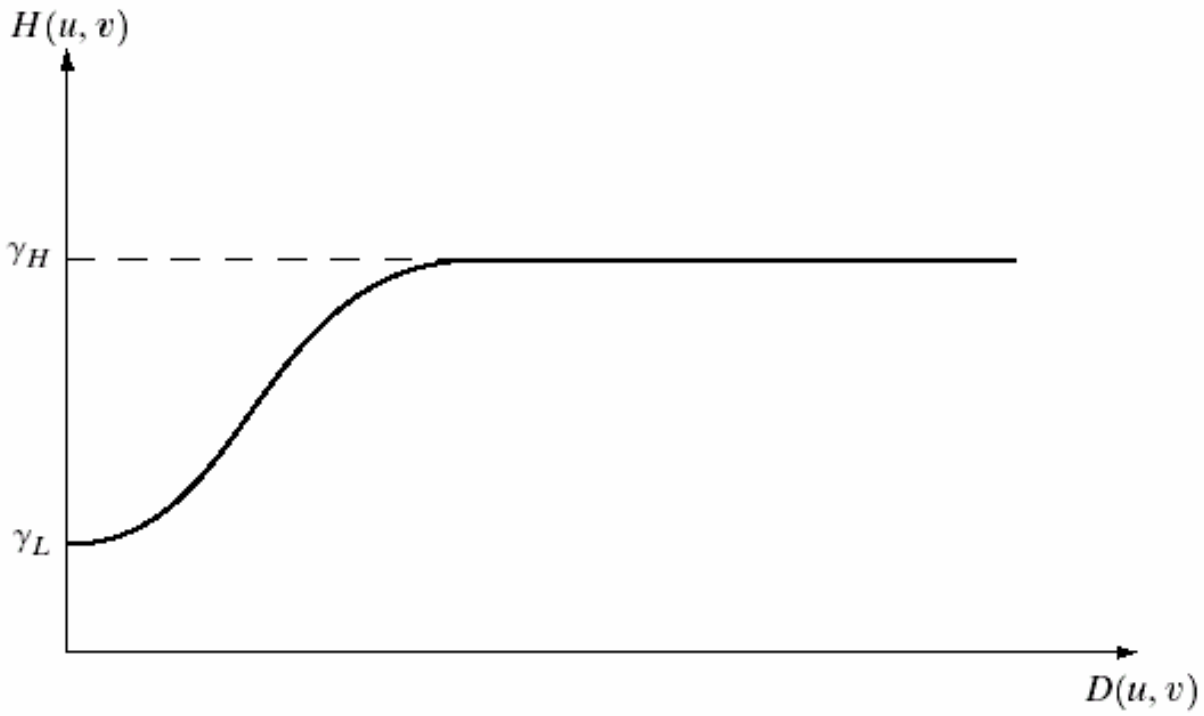
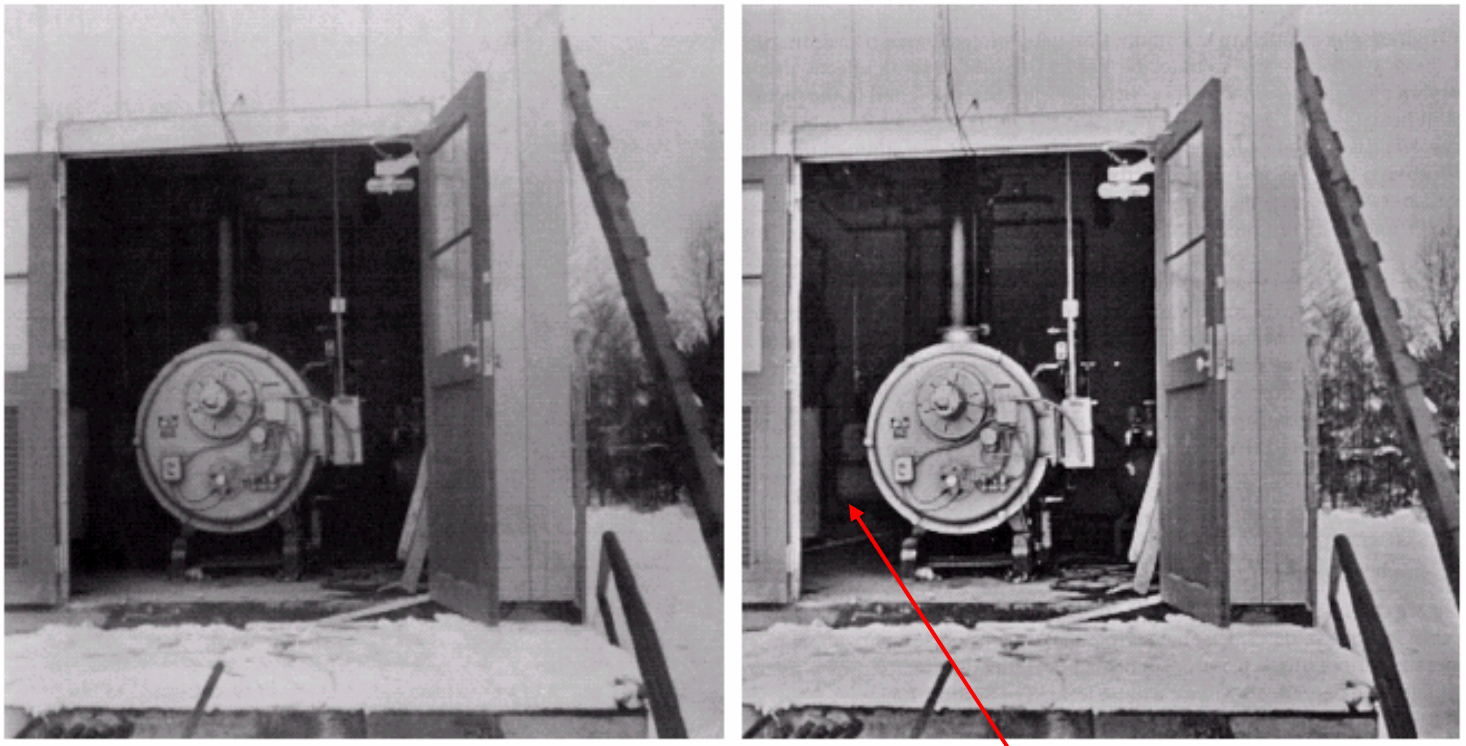


FIGURE 4.32
Cross section of a circularly symmetric filter function. $D(u, v)$ is the distance from the origin of the centered transform.

Homomorphic Filtering

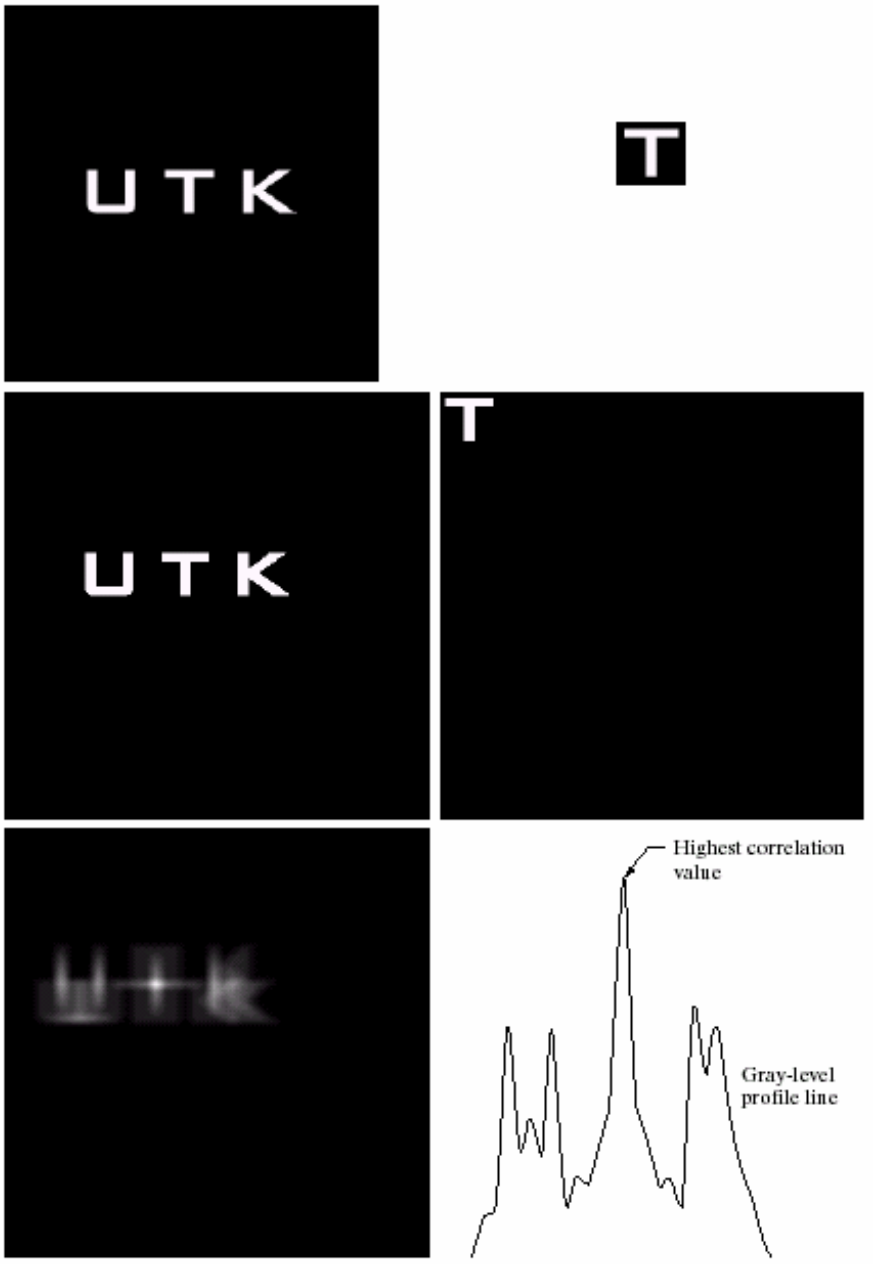
a b

FIGURE 4.33
(a) Original image. (b) Image processed by homomorphic filtering (note details inside shelter). (Stockham.)



More details in the room can be seen!

Correlation Application: Object Detection



a b
c d
e f

FIGURE 4.41
(a) Image.
(b) Template.
(c) and
(d) Padded images.
(e) Correlation function displayed as an image.
(f) Horizontal profile line through the highest value in (e), showing the point at which the best match took place.